The World in a Point: Euclid's Alexandrian Engagement with Philosophy and Poetry

Although scholars acknowledge the innovations in the poetry and mathematics of the Hellenistic period, very rarely do they make any direct connection between them. Building on the work of Netz 2009, who has argued that Archimedes exhibits a playful, learned style similar to that of the Hellenistic poets, I focus on Euclid, Archimedes' influential elder and an Alexandrian contemporary of Callimachus, Theocritus, and Aratus. Euclid's similarity is not in style but in content and form. The Definitions in *Elements* I assert Euclid's contribution to an ongoing dialogue with the past much like the programmatic poetry of his contemporaries. The first definition of a point specifically engages with philosophical discussions of whether one can explain the existence of a point. Euclid explains its existence in the following definitions, I argue, by constructing an implicit Hesiodic cosmogony, a form often adapted in contemporary didactic poetry.

Netz calls the opening of the Elements "the most impersonal form of the Greek mathematical introduction" (p. 98). While Euclid does not use the first person, his word-choice in the first definition asserts a programmatic tone: "A point (σημεῖον) is that of which there is no part" (Def. 1). Before Euclid, however, the standard mathematical word for point was στιγμή (Heath 1926). In fact, only Aristotle uses σημεῖον before Euclid in this mathematical sense, and always in the context of stressing that a point cannot be explained, but must necessarily be inferred (Arist. *Ph*. 240b3 and *APo*. 76b5). Aristotle reacts against Plato's explanation for the existence of a στιγμή (Arist. *Met*. 992a20-22 cf. Ross 1953), which literally means "puncture," and so uses σημεῖον, which he uses elsewhere for anything probable versus provable (*Pr*. 70a11 and *R*h. 1357a33).

Thus it is bold when Euclid begins his work with this word and not manifestly clear why he would want to do so. The following definitions make it clear that Euclid is not merely defining elements that exist in the mathematical world, but ones that make that world exist: "a line is length without breadth" (Def. 2); "the extremities (πέρατα) of lines are points" (Def. 3); "a straight line is that which lies evenly with the points on itself" (Def. 4); "a surface is that which has length and breadth only" (Def. 5); "the extremities of a surface are lines" (Def. 6); and "a plane surface is that which lies evenly with the straight lines on itself" (Def. 7). Once a plane is defined, Euclid can then define angles, circles, triangles and quadrilaterals. (Def. 8-23). What makes all these basic shapes of geometry possible, in Euclid's *schema*, is the plane surface, which is made up of lines, which are made of points. The word for "surface" is ἐπιφάνεια. While this is a common word for "surface" or "plane," it takes on special significance here because of Euclid's choice for "point" in the first definition: a portent (σημεῖον) leads to an epiphany (ἐπιφάνεια).

Euclid starts with a word for point that Aristotle used to make it an abstract concept, only to show that it is what is behind an ἐπιίπεδος ἐπιφάνεια (Def. 7), literally something that you can see, unlike the line and point, but also step upon to prove its existence. Euclid has made a point the beginning of a cosmogony that produces the world of geometry. Similarly, Hesiod does not simply list gods in his Theogony, but explains them in an order that also explains the origins of the visible world. Hesiod's strategy proved to remain profoundly influential, particularly in Hellenistic poetry. Callimachus' interest in origins in the *Aetia*, for example, leads him to acknowledge Hesiod and the beginning of the *Theogony* as his model (Call. Aet. I. fr. 2. 2-3).

Proclus, the fourth-century Neoplatonist, fully understood the significance of Euclid's use of the word σημεῖον in his commentary of Euclid's first book (Morrow 1970). Other than

Proclus, however, it seems that this word lost its significance in mathematics very quickly, ironically because of Euclid's influence: it becomes the simple word for "point" in Archimedes and later Greek mathematicians. In Latin, it is always translated as *punctum*. It is important to look at Euclid's active manipulation of language and form, for by thinking of the *Elements* simply as "math," instead of as a product of the rich Alexandrian culture that was engaged with past philosophical and literary traditions, we miss a major aspect of Euclid's importance.

Works Cited

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